1. T(n) = T(n-2) + 2n, T(0)=0

= T(n-4) + 2n-2 + 2n

= T(n-6) + 2n-4 + 2n-2 + 2n

= T(n-8) + 2n-6 + 2n-4 + 2n-2 + 2n

.

.

.

= T(n-2\*k) + 2n-2(k-1) + 2n-2(k-2) +...+ 2n-2 + 2n

= T(n-2\*k) +

Let,

n-2\*k = 0

n = 2k

k = n/2

So,

T(n) = T(0) +

= 0 +

= 0 +

= 0 +

= 0 +

= Θ(2n)

1. T(n) = (T(n-2))2, T(0)=2

= (T(n-4)2)2

= (T(n-4))4

= (T(n-6)2)4

= (T(n-6))8

= (T(n-8)2)8

= (T(n-8))16

.

.

.

= (T(n-2\*k))^

Let,

n-2\*k = 0

n = 2k

k = n/2

So,

T(n) = (T(0))

= (2)

= (2)

= Θ()

1. T(n) = T(√n) + Θ(log2n), T(2)=1  
    = T(n1/2) + clog2n  
    = (T(n1/4) + clog2n1/2) + clog2n [Substitution]  
    = T(n1/4) + (c/2)log2n + clog2n  
    = (T(n1/8) + clog2n1/4) + (c/2)log2n + clog2n [Substitution]  
    = T(n1/8) + (c/4)log2n + (c/2)log2n + clog2n  
    .

.

.

= T(n1/2^k) + clog2n  
Let,

n1/2^k = 2

lg2n1/2^k = lg22

1/2klg2n = 1

lg2n = 2k

lg2lg2n = lg22k

lg2lg2n = k

So,

T(n) = T(2) + clog2n

= 1 + clog2n

= O(lg2n)

1. T(n) = T(n/2) + T(√n) + Θ(n), T(1)=1  
   T(n) ≤ 2T(n/2) + cn  
    ≤ 2(2T(n/4) + cn/2) + cn [Substitution]  
    ≤ 4T(n/4)+ cn + cn  
    ≤ 4(2T(n/8) + cn/2) + cn + cn [Substitution]  
    ≤ 8T(n/8) + cn + cn + cn  
    .

.

.  
≤ 2kT(n/2k) + kcn

Let,

n/2k = 1

n = 2k

lg2n = lg22k

lg2n = k

So,

T(n) = 2lg2\_nT(1) + lg2n\*cn

= 2lg2\_n\*1 + lg2n\*cn

= O(nlg2n)

1. T(n) = 2T(n/3) + Θ(nlg2n), T(1)=1  
    = 2T(n/3) + cnlg2n  
    = 2(2T(n/9) + cn/3lg2n/3 ) + cnlg2n [Substitution]  
    = 4T(n/9) + 4cn/3lg2n/3 + cnlg2n  
    = 4(2T(n/27) + cn/9lg2n/9) + 4cn/3lg2n/3 + cnlg2n [Substitution]  
    = 8T(n/27) + 8cn/9lg2n/9 + 4cn/3lg2n/3 + ccnlg2n  
    .

.

.

= 2kT(n/3k) +

Let,

n/3k = 1

n = 3k

lg3n = lg33k

lg3n = k

So,

T(n) = 2lg3\_nT(1) +

= 2lg3\_n\*1 +

= 2lg3\_n +

= 2lg3\_n +

= nlg3\_2 +

= Θ(nlg2n)

1. T(n) = T(n/5) + Θ((lg2n)2), T(1)=1  
    = T(n/5) + c(lg2n)2 = T(n/52) + c(lg2n/5)2 + c(lg2n)2 [Substitution]  
    = T(n/53) + c(lg2n/52)2 + c(lg2n/5)2 + c(lg2n)2 [Substitution]  
    .

.

.

= T(n/5k) + c

Let,

n/5k = 1

n= 5k

lg5n= lg55k lg5n= k

So,

T(n) = T(1) + c

= 1 + c

= 1 + c

= 1 + c

= Θ((lg2n)2)

1. T(n) = T(n/3) + T(2n/6) + Θ(n), T(1)=1  
    = T(n/3) + T(n/3) + Θ(n)  
    = 2T(n/3) + Θ(n)

= 2T(n/3) + cn

= 2(2T(n/9) + cn/3) + cn [Substitution]

= 4T(n/9) + 2cn/3 + cn

= 4(2T(n/27) + cn/9) + 2cn/3 + cn [Substitution]

= 8T(n/27) + 4cn/9 + 2cn/3 + cn

.

.

.

= 2kT(n/3k) + cn

Let,

n/3k = 1

n = 3k

lg3n = lg33k

lg3n = k

So,

T(n) = 2lg3\_nT(1) + cn

= nlg3\_2\*1 + cn

= nlg3\_2 + cn \* (1-(2/3)lg3\_n)/(1-2/3)

= n0.63 + cn \* (1-(2/3)lg3\_n)/(1-2/3)

= O(n)